When predicting returns from existing factor returns, GMAM 3.0 models the predicted de-smoothed returns at each point in time as:

where is a length vector of factor returns including an intercept dummy, is the exposure, is the predicted de-smoothed return, is the risk-free rate of return, and is the error. GMAM 3.0 uses Markov Chain Monte Carlo (MCMC) to compute estimates of β and the distributional parameters of . To create a point estimate, the provided returns are averaged across draws in the MCMC process. Specifically, the expected predicted de-smoothed returns are estimated from N simulations as:

Here, is derived from model parameters sampled at each iteration and is a t-distribution degree of freedom parameter likewise sampled. The average of the idiosyncratic error terms tend towards zero, given a large number of draws[[1]](#footnote-1).

Re-smoothed returns mimic the underlying characteristics of the reported returns to a greater degree than the de-smoothed returns at the cost of not reflecting the true economic changes in value over time. The factor de-smoothed returns are computed first by computing the de-smoothed returns as previously described. Then the re-smoothed returns are calculated as:

where is a vector of smoothing coefficients derived from estimated model parameters, is orthogonal idiosyncratic error (measurement error) drawn from a normal distribution of variance , and is a vector of contemporaneous and *lagged,* de-smoothed returns. As with the de-smoothed returns, plug-in variance estimates of the average will underestimate the volatility of the series. Estimates of volatility derived from synthetic returns should therefore incorporate the idiosyncratic error variance , for which estimates can be derived directly from model output and are available upon request.

1. This is not generally important when assessing the risk since the average idiosyncratic risk itself can be directly read from the model outputs. However it does imply that plug-in volatility estimates of the empirical average of (e.g. will, in accordance with the central limit theorem, underestimate the idiosyncratic volatility since . To compensate for this, direct estimates of along with other related volatility metrics are available upon request. [↑](#footnote-ref-1)